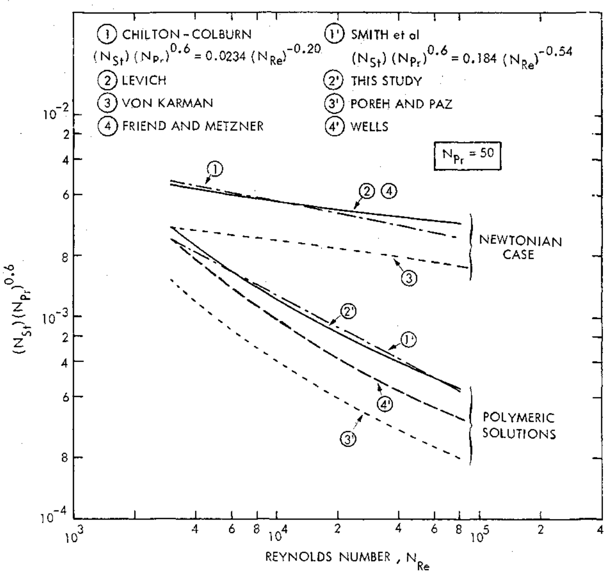

Fig. 3 Heat-transfer characteristics when $N_{Pr} = 50$.

give us some indication about the reliability of these models; as conditions of maximum drag reduction are well defined.

In Fig. 1 comparisons between the different models when Prandtl numbers are 5 and 50 were done. It seems that when $N_{Pr} = 5$ the model suggested in this study is closer to the experimental results than the other models; however, the differences in the values of Nusselt numbers predicted by the different models are not large. In the case of $N_{Pr} = 50$ these differences are much larger, the suggested model of this study is close to Chilton-Colburn curve as well as to Smith et al. curve. The Wells model is less close and the Poreh and Paz model is very far from both of these curves. The values of the friction coefficient in the Newtonian case as well as maximum drag reduction were shown too in this figure.

In Figs. 2 and 3 the values of $(N_{St})(N_{Pr})^{0.6}$ as a function of Reynolds number according to the different models are shown. Also from these figures it seems that the model suggested in this study can describe quite accurately heat-transfer phenomena for quite wide ranges of Prandtl as well as Reynolds numbers.

References

[1] Meyer, W. A., "A Correlation of the Frictional Characteristics for Turbulent Flow of Dilute Non-Newtonian Fluids in Pipes," *AIChE Journal*, Vol. 12, 1966, pp. 522–525.

[2] Elata, C., Lehrer, J., and Kahanovitz, A., "Turbulent Shear Flow of Polymer Solutions," *Israel Jounal of Technology*, Vol. 4, No. 1, 1966, pp. 87–95.

[3] Van-Driest, E. R., "The Damping of Turbulent Flow by Long Chain Molecules," Scientific Rept. AFOSR 67-2369, 1967, Ocean System Operations, North American Rockwell Corp., Anaheim, Calif.

[4] Seyer, F. A. and Metzner, A. B., "Turbulence Phenomena in Drag Reducing Systems," presented at 60th Annual AIChE Meeting, New York, 1967; also *AIChE Journal*, Vol. 15, No. 3, May 1969, pp. 426–434.

[5] Fabula, A. G., "Attainable Friction Reduction on Large Fast Vessels," Technical Paper 123, 1969, Naval Undersea Warfare Center, Pasadena, Calif.

[6] Virk, P. S., "The Toms Phenomenon-Turbulent Pipe Flow of Dilute Polymer Solutions," Ph.D. thesis, 1966, MIT, Cambridge, Mass.

[7] Smith, K. A., Keuroghlian, G. H., Virk, P. S., and Merril, E. W., "Heat Transfer to Drag-Reducing Polymer Solutions," *AIChE Journal*, Vol. 15, No. 2, March 1969, pp. 294–297.

[8] Paterson, R. W. and Abernathy, F. H., "Turbulent Flow Drag Reduction and Degradation with Dilute Polymer Solutions," *Journal of Fluid Mechanics*, Vol. 43, Pt. 4, 1970, pp. 689–710.

[9] Poreh, M. and Paz, U., "Turbulent Heat Transfer to Dilute Polymer Solutions," *International Journal of Heat and Mass Transfer*, Vol. 11, 1968, pp. 805–818.

[10] Von-Kármán, T., "The Analogy between Fluid Friction and Heat Transfer," *Transactions of the ASME*, Vol. 61, 1939, p. 705.

[11] Wells, C. S., "Turbulent Heat Transfer in Drag Reducing Fluids," *AIChE Journal*, Vol. 14, No. 3, May 1968, pp. 406–410.

[12] Friend, W. L. and Metzner, A. B., "Turbulent Heat Transfer Inside Tubes and the Analogy among Heat Mass and Momentum Transfer," *AIChE Journal*, Vol. 4, No. 4, Dec. 1958, pp. 393–402.

[13] Levich, V. G., *Physicochemical Hydrodynamics*, Prentice-Hall, Englewood Cliffs, N.J., 1962, pp. 139–157.

[14] Rubin, H., "Estimation of Mass Diffusion Reduction by Drag Reducing Polymeric Additives," *Journal of Hydronautics*, Vol. 5, No. 2, April 1971, pp. 73–75.

Drag Reduction by Use of MHD Boundary-Layer Control

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1. Introduction

DRAG reduction for fully submerged vehicles has received considerable attention during the past decade.[1] The addition of polymer additives to the fluid in the boundary layer, preventing transition to a turbulent boundary layer by means of area or slot suction, and the optimization of vehicle shape are the methods of drag reduction that have been studied. Application of these methods has not been made, however, because of the practical problems associated with each of the proposed methods.

The method proposed here is similar to the suction method in that the boundary profile is altered to prevent transition to a turbulent boundary layer. The boundary-layer control would be effected by utilizing the Lorentz force that may be made to act on a conducting fluid particle moving in a magnetic field. An optimum magnetic field distribution will be sought to maintain a laminar boundary layer, thus achieving drag reduction and noise reduction at the same time.

Idealized problems of the motion of conducting fluids in the presence of external magnetic fields have been examined by many investigators with various inconclusive results. Hartmann and Lazarus[2] found that the transverse magnetic field was to increase the pressure gradient for the channel flow of mercury which was originally laminar, but increase of field strength produced a decrease in pressure gradient up to a certain point for the flow which was originally turbulent. Murgatroyd[3] was able to suppress turbulence at Reynolds number of 10^5 by applying a transverse magnetic field to a channel flow of a mercury. Harris[4] extended the dimensional analysis in hydrodynamic flows to the magneto-fluid-mechanic channel flow utilizing the experimental data of Hartmann and Lazarus, and Murgatroyd. Rassow[5] studied the laminar boundary-layer flow of ionized gas over a flat plate in the presence of a transverse magnetic field and found that the skin friction was reduced but the total drag increased.

Our discussion will be mainly focused upon the submerged vehicles under seawater. The momentum integral

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method is used for its simplicity and relative accuracy as noted by Heywood and Moffatt.[6] The velocity profiles are approximated by a one parameter family, combining the Blasius profile and the asymptotic suction profile. The choice of suction profile should be quite good since the work by Lykoudis[7] indicated the similarity between sucking effect to the ordinary fluid mechanic boundary layer and the transverse magnetic field effect to the magneto-fluid mechanic boundary layer for a large Hartmann number.

2. Boundary-Layer Analysis

It is assumed that a semi-infinite flat plate is moving through a conducting incompressible fluid with a constant velocity $\mathbf{U}_\infty$ parallel to its own plane. *DC* magnetic field lines are applied perpendicular to freestream velocity to just maintain the laminar boundary layer on the verge of transition. The electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ are related by $\mathbf{E} = -\mathbf{U}_\infty \times \mathbf{B}$, thus limiting the Lorentz force effect to the boundary layer. The regular magnetohydrodynamic approximations[8] are also used here.

By incorporating the assumptions given above we can simplify the fundamental equations, consisting of Maxwell's equations, incompressible form of modified Navier-Stokes equation, equation of continuity, and generalized Ohm's law.[9]

After some simplification the component equations are reduced to

$$u\partial u/\partial x + v\partial u/\partial y + \sigma B^2/\rho(u - U_\infty) = \nu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2) \quad (1)$$

$$u\partial v/\partial x + v\partial v/\partial y = \nu(\partial^2 v/\partial x^2 + \partial^2 v/\partial y2) \quad (2)$$

Since we are mainly interested in boundary-layer flow, the appropriate boundary-layer approximations[10] can be applied to the above equations to obtain

$$u\partial u/\partial x + v\partial u/\partial y + \sigma B^2/\rho(u - U_\infty) = \nu\partial^2 u/\partial y^2 \quad (3)$$

In addition, we also have the equation of continuity

$$\partial u/\partial x + \partial v/\partial y = 0 \quad (4)$$

and the boundary conditions

$$\begin{aligned} y = 0: &\quad u = 0, \quad v = 0 \\ y = \infty: &\quad u = U_\infty, \quad v = \partial u/\partial y = 0 \end{aligned} \quad (5)$$

In the Eqs. (1–5), x is the distance along the plane; y is the distance perpendicular to the plane; u and v are the velocity components in the boundary layer, parallel and perpendicular to the wall; σ is the conductivity of fluid; ρ is the density of fluid; ν is the kinematic viscosity.

The momentum integral method[10] is applied to solve these boundary-layer equations. The first relationship is obtained from Eqs. (3) and (4), and the second relationship is obtained by applying the streamwise component of a momentum equation at the wall. These two equations are then nondimensionalized with the aid of the freestream velocity U_∞ and a constant length of reference L. The velocity profiles in the boundary layer are approximated by a one parameter family.[1] The final forms of the two relationships are

$$dR_\theta/d\bar{x} + H^2[1 - K(2 - 6/\pi)]R_\theta/R_L g(k) = g(K)[1 + K(1 - \pi/6)]R_L/R_\theta \quad (6)$$

$$H^2 = (1 + K)g(K)^2 R_L{}^2/R_\theta{}^2 \quad (7)$$

where $R_\theta = U_\infty\theta/\nu$ is the Reynolds number based on the momentum thickness and freestream velocity; $R_L = U_\infty L/\nu$ is the Reynolds number based on the reference length L and freestream velocity; $H = BL(\sigma/\eta)^{1/2}$ is the Hartmann number; K is the velocity profile shape factor; $\bar{x} = x/L$ and $g(k) = 0.5 + 0.06656K - 0.02358K^2$.

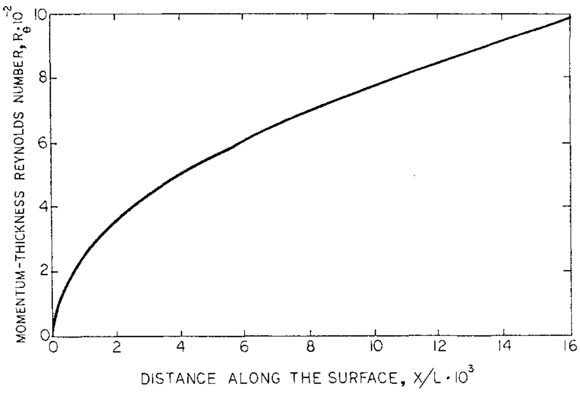

Fig. 1 Momentum-thickness Reynolds number along the surface.

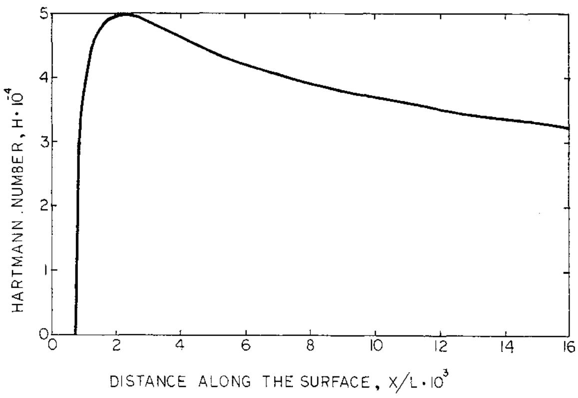

Fig. 2 Hartmann number distribution along the surface.

3. Numerical Solution

Equations (6) and (7) are two equations for three unknowns; $R_\theta(\bar{x})$, $H(\bar{x})$ and $K(\bar{x})$. There is no need for applied magnetic field near the leading edge at $\bar{x} = 0$, where we should have a laminar boundary layer. The Eqs. (6) and (7) can then be solved simultaneously for R_θ and K, with H set equal to zero. This solution should be the Blasius solution. In this domain the boundary layer is subcritical up to a point $\bar{x}_c$ where R_θ attains the critical value. From that point onwards, calculation of the boundary-layer parameters are performed to maintain a critical boundary layer. For this condition we can use the graph obtained by Tetervin and Levine,[11] giving the critical momentum-thickness Reynolds number R_{θ_c} as a function of K, which provides the third equation for the three unknowns.

Equations (6) and (7) can be combined to eliminate H and give the following differential equation

$$dR_\theta/d\bar{x} = (R_L/R_\theta)g(K)[K(2 - \pi/6 - 6/\pi) + K^2(2 - 6/\pi)] \quad (8)$$

with the initial condition

$$\bar{x} = \bar{x}_c = 7.4 \times 10^{-4}: \quad R_\theta = 225, \quad H = 0, \quad K = -1 \quad (9)$$

which is corresponding to $U_\infty = 80$ fps, $L = 20$ ft, and $\nu = 10^{-5}$ ft^2/sec. The Runge-Kutta method was used to integrate Eq. (8) numerically in conjunction with the graph relating R_{θ_c} to K. The computer results are summarized in Fig. 1 and Fig. 2.

4. Summary

Momentum-thickness Reynolds number increases asymptotically with distance as expected, but the rate of increase is rather slow. Hartmann number, which is proportional to magnetic field, increases very rapidly from zero at $\bar{x} = 7.4 \times 10^{-4}$ to about 5×10^4 at $\bar{x} = 2.13 \times 10^{-3}$, then drops off very slowly toward zero. This characteristic is also expected from its analogy to suction case. Although the rather rapid change of Hartmann number near the leading edge might jeopardize our approximate analysis, the order of magnitude obtained should be useful to evaluate the efficiency of magnetohydrodynamic boundary-layer control.

When seawater is used as an example the required maximum magnetic flux density is about 1.5×10^6 Gauss. Magnetic field of this order may be practical with recent advancement in superconducting magnet and additional incentive given by noise reduction effect. The magnetic Reynolds number is found as $R_m = \mu\sigma U_\infty L = 5.6 \times 10^{-4}$, which is much smaller than unity and our assumption as to neglect the induced magnetic field is valid.

One basic problem which is not studied here is the relationship between Hartmann effect and turbulence-damping effect, which should be significant at a large magnetic field. Detailed account of the derivations leading to the equations given in this paper can be found elsewhere.[12]

References

[1] Anderson, G. F., Murthy, V. S., and Sutera, S. P., "Laminar Boundary-Layer Control by Combined Blowing and Suction in the Presence of Surface Roughness," *Journal of Hydronautics*, Vol. 3, No. 3, July 1969, pp. 145–151.

[2] Hartmann, J. and Lazarus, F., "Hg-Dynamics II-Experimental Investigations on the Flow of Mercury in a Homogeneous Magnetic Field," *Kongelige Danske Videnskabernes Selskab, Det Matematsik-Fysiske Meddelelser*. Vol. 15, Nos. 6–7, 1937.

[3] Murgatroyd, W., "Experiments on Magneto-Hydrodynamic Channel Flow," *Philosophical Magazine*, Vol. 44, No. 359, Dec. 1953, pp. 1348–1354.

[4] Harris, L. P., *Hydromagnetic Channel Flows*, MIT Press, Cambridge, Mass., 1960.

[5] Rossow, V. J., "On Flow of Electrically Conducting Fluids over a Flat Plate in the Presents of a Transverse Magnetic Field," Rept. 1358, 1958, NASA.

[6] Heywood, J. B. and Moffatt, W. C., "Validity of Integral Methods in MHD Boundary-Layer Analysis," *AIAA Journal*, Vol. 3, No. 8, Aug. 1965, pp. 1565–1567.

[7] Lykoudis, P. S., "Transition from Laminar to Turbulent Flow in Magneto-Fluid Mechanic Channels," *Reviews of Modern Physics*, Vol. 32, No. 4, Oct. 1960, pp. 796–798.

[8] Pai, S., *Magnetogasdynamics and Plasma Dynamics*, Springer-Verlag, Wien, Austria, 1962.

[9] Carrier, G. F. and Greenspan, H. P., "The Magnetohydrodynamic Flow Past a Flat Plate," *Journal of Fluid Mechanics*, Vol. 6, Pt. 1, 1959, pp. 77–96.

[10] Schlichting, H., *Boundary Layer Theory*, McGraw Hill, New York, 1960, Chapts. 7 and 8.

[11] Tetervin, N. and Levine, D. A., "A Study of the Stability of the Laminar Boundary Layer as Affected By Changes in the Boundary-Layer Thickness in Regions of Pressure Gradient and Flow through the Surface," TN2752, Aug. 1952, NACA.

[12] Anderson, G. F. and Wu, Y. K., "Drag Reduction by Use of MHD Boundary Layer Control," Rept. 1, May 1970, Southeastern Massachusetts Univ., North Dartmouth, Mass.